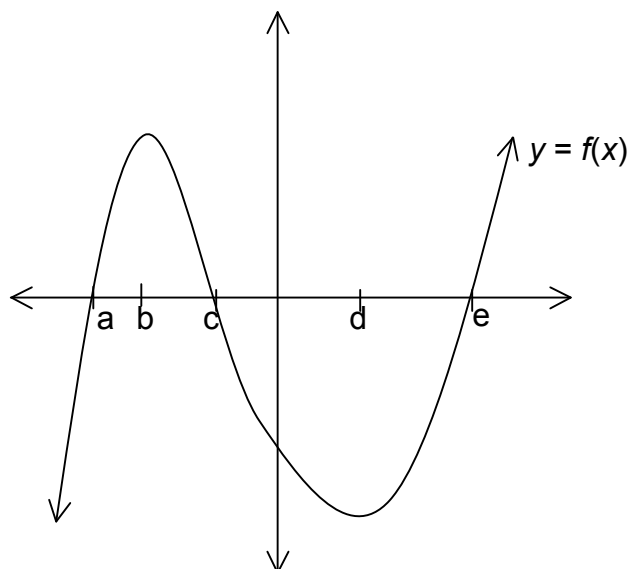


## 10.2 Concavity: Points of Inflection

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1) Consider the graph of the function  $y = f(x)$ .



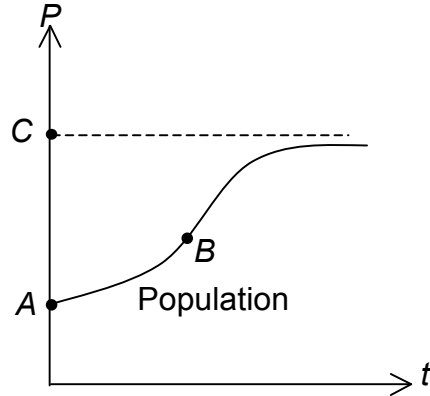
- a) A curve is said to be *concave up* on an interval  $(x_1, x_2)$  if at each point on the interval the curve is above its tangent at the point. On which interval(s) is  $f(x)$  concave up?
- b) A curve is said to be *concave down* on an interval  $(x_1, x_2)$  if at each point on the interval the curve is below its tangent at the point. On which interval(s) is  $f(x)$  concave down?
- c) Over what interval(s) are the slopes of the tangent lines increasing? decreasing?

Assume that the first and second derivatives of function  $f$  exist. If  $f''(x) > 0$  on an interval  $I$ , the graph of  $f$  is **concave up** on the interval. If  $f''(x) < 0$  on an interval  $I$ , the graph of  $f$  is **concave down** on the  $I$ . We also say that the graph of  $y = f(x)$  is concave up at  $(a, f(a))$  if  $f''(a) > 0$  and that the graph is concave down at  $(b, f(b))$  if  $f''(b) < 0$ . The point where concavity changes is called a **point of inflection**.

- 2) Use the first derivative to locate any relative extremes. Use the second derivative to determine intervals where the function is concave upward, to determine intervals where it is concave downward, and to locate points of inflection. Sketch the graph of the function.

$$f(x) = x^3 - 9x^2$$

- 3) The following figure shows the growth of a population as a function of time.



- a) If  $P$  represents the population and  $t$  represents the time, write a symbol that represents the rate of change (growth rate) of the population with respect to time.
- b) Which of  $A$ ,  $B$ , and  $C$  corresponds to the point at which the growth *rate* attains its maximum?
- c) Which of  $A$ ,  $B$ , and  $C$  corresponds to the upper limit of the population?

- 4) Suppose that a company's daily sales volume attributed to an advertising campaign is given by

$$S(t) = \frac{3}{t+3} - \frac{18}{(t+3)^2} + 1$$

- a) Find how long it will be before sales volume is maximized.
- b) Find how long it will be before the rate of change of sales volume is minimized. That is, find the point of diminishing returns.